The Analemma Dilemma

Solving visualisation issues in astronomy using 3D graphics

Hannah Bull

This project focuses on visualisation problems involved in modelling astronomical phenomena, with particular reference to how the analemma can be explained and depicted using graphical figures. By considering various factors and adjustments, the simulated analemma can be graphed with high accuracy through computer simulations. These are illustrated with two example locations in Singapore and Athens.

The motion of the sun relative to the earth is a result of the combined effects of the rotation of the earth and the revolution of the earth around the sun. The rotation axis of the earth is tilted at approximately 23.45 degrees to the plane of the earth's orbit around the sun, and the revolution of the earth around the sun follows an elliptical path with eccentricity 0.0167. These irregularities in revolution and rotation allow for variations in the length of daylight and changes in sunrise and sunset times. The path of the sun at a specific time and location over a period of one year is referred to as the analemma. This phenomenon can be easily visualized and detected from the earth; the only difficulty is that it requires a whole year to do so. Figure 1 shows one such photographic record of the analemma, in which a photograph of the sun has been taken at 8:30am each morning.

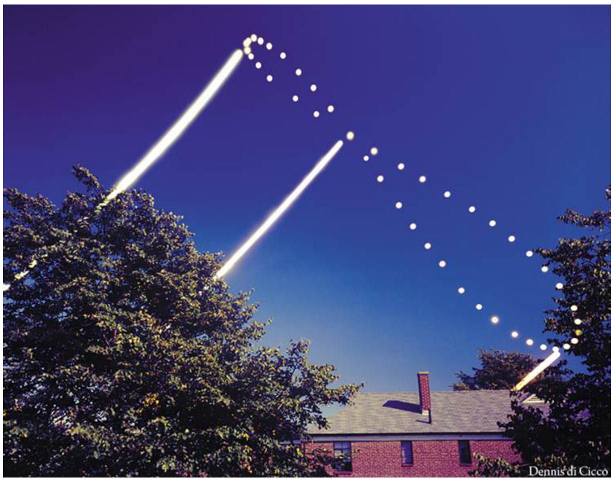


Figure 1. 8:30am analemma at 46°32'N (di Cicco, 1978-9).

Modelling the analemma requires the ability to determine the position of the sun in the sky when given a time, date, and location. Graphical representations of the analemma are important, as it is not a phenomenon that can be seen at any single moment in time. The use of such graphics serves as a means of explaining related and more familiar phenomena, such as why the sun rises and sets at different times, even on the equator, and how sundials can tell the date as well as the time. To model the analemma, we use Mathematica 7.0. The calendrical and astronomical algorithms required for these calculations come from the package Calendrica, written in Lisp by Edward M. Reingold and Nachum Dershowitz and converted for use in Mathematica by Robert C. McNally (2009).

A difficulty arises in accurately depicting the analemma in programmable figures, such that the viewer is easily able to understand the phenomenon. Problems that arise from the depiction of the analemma are also common to models of other astronomical phenomena. Good astronomical graphics strive toward realism and scientific accuracy, although strategic modifications to scale and viewing position are often unavoidable. An external view of the universe is useful for depicting large-scale solar motions, but is limited in its applications for usefully depicting the analemma. Corrections in viewpoint allow for the creation of graphical models, which resemble real photographs of the analemma. It is possible to create both realistic and accurate models of the analemma, allowing for better visualisation and understanding of this phenomenon. It is useful to model the analemma using programmable figures, as this will allow for easy adaptation of models for the analemma as seen at different latitudes and times of day.

Generally, celestial bodies are either projected onto a sphere, such as the celestial sphere, or onto a plane by using one of numerous spherical projections. The earth lies at the centre of the celestial sphere and stars, planets, and other bodies are projected onto the sphere itself. This is useful in explaining many phenomena but is also an unrealistic depiction of space. A typical celestial sphere model is depicted in Figure 2 below.

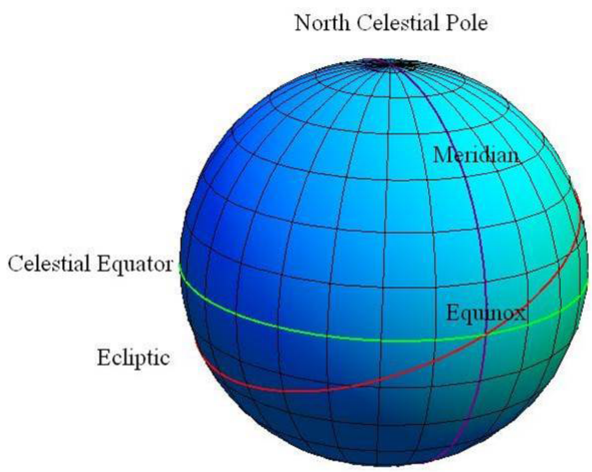


Figure 2. Celestial Sphere Model.

The celestial equator is a projection of the earth's equator onto the celestial sphere. Similarly, latitudinal and longitudinal lines on the celestial sphere correspond to the projection of the earth's latitudinal and longitudinal lines. The ecliptic traces the apparent path of the sun throughout the year and lies at an angle of approximately 23.45 degrees to the celestial equator due to the tilt of the earth's axis. The points where the sun crosses the celestial equator correspond to the vernal and spring equinox. On these dates at a location on the equator, the sun can be seen directly overhead.

Although this image of the imaginary celestial sphere is practical for explaining the apparent path of the sun, the first fundamental problem with this diagram is that nobody has even witnessed the sun's motion from this angle. The motion of the sun across the sky over one year, as seen from the earth, is dependent on latitude and appears very different from this depiction based on an external perspective. Another significant problem with this image is that it can be easily confused with the apparent motion of the sun throughout the day. This model of the celestial sphere also does not display another key factor contributing to the analemma's shape – the eccentricity of the earth's orbit around the sun.

While it is very easy to place the analemma on the celestial sphere in this manner, it is also not very useful in explaining the phenomenon. The shape and angle of the analemma are dependent on the latitude of the viewer and the time of day. Figure 3 shows the appearance of the analemma for a viewer at 3pm at 20 degrees north, projected onto the celestial sphere. Projecting an image of the analemma at a given latitude and time onto a celestial sphere is misleading, because it implies that the analemma has a fixed angle and location in the sky for all viewers. Furthermore, it may be confusing to note that in such diagrams the celestial sphere is not of the same orientation of the celestial sphere as shown in Figure 3. The celestial equator is no longer marked; rather, the horizon is for a viewer at 20 degrees latitude at a given time of day.

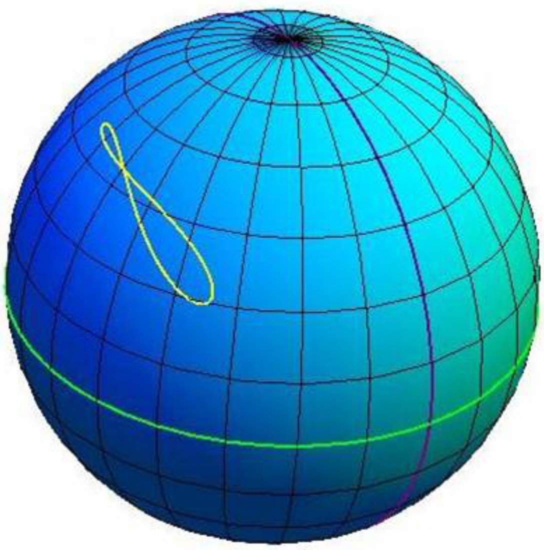


Figure 3. 3pm at 20°N analemma on a sphere (Teo, 2002).

Explanations of the analemma using an external view of the celestial sphere are, as a result of these issues, quite limited. In order to produce useful images of the analemma, it is essential to attempt to depict the analemma realistically. A more effective means of depicting the analemma is to 'open up' the celestial sphere and to view the analemma and the celestial sphere from the inside, as it would be seen from earth.

Explanation of the Analemma through Programmable Figures

Depictions of the Sun's Declination

The declination of the sun is the angle it makes with the plane of the celestial equator. At the equinoxes, the sun's declination is 0 degrees and at the summer and winter solstices, maximum and minimum declinations are reached at ±23.45 degrees. The declination of the sun can be visualized by comparing the sun's path along the ecliptic with a path along the celestial equator. The mean sun refers to a fictional sun that travels across the celestial equator. This is equivalent to the sun's motion if the earth were not tilted and if its revolution around the sun were circular. The true sun follows the path of the ecliptic, tilted at 23.45 degrees to the celestial equator.

The graph of the declination angle δ of the sun over one year is a sine function involving the sun's maximum and minimum declination and the true sun's longitudinal position λ (in degrees) on the celestial sphere. The sun's longitudinal position may be extracted from a table or can be reproduced from astronomical algorithms (Meeus, 1998).

$$\sin\delta = 23.45\sin\lambda$$
$$\Rightarrow \delta = \sin^{-1}\left(23.45\sin\lambda\right) \text{ (in degrees)}$$

Plotting the declination angle δ against the true sun's longitudinal position λ produces the following graph, where the horizontal axis is the path of the mean sun ($\delta = 0$).

Figure 4 is a projection of the ecliptic and the celestial equator onto a two-dimensional plane. At the times of the summer and winter solstices, the sun reaches its northernmost and southernmost extremes respectively, as is clearly depicted by the two-dimensional graphic.

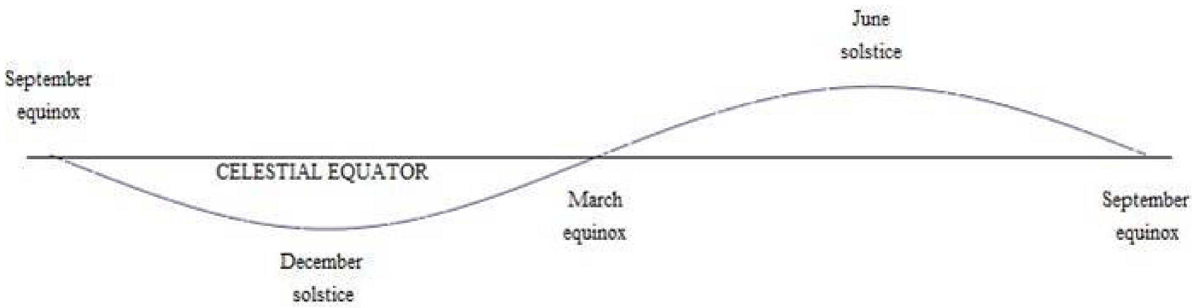


Figure 4. Path of the true sun (Teo, 2002).

This diagram is a clear and simple representation of the path of the true sun along the ecliptic as compared with the path of the mean sun along the celestial equator; however, the lengths are incorrectly represented. By projecting this three-dimensional phenomenon onto a two-dimensional plane, the relative lengths of the celestial equator and the ecliptic become incorrect. The path of the true sun appears to be longer than the path of the mean sun and it seems that the true sun must travel further and faster than a sun following the celestial equator, although from the three-dimensional figure of the celestial sphere in section 1, it is clear that the path of the celestial equator is no longer than the ecliptic. The two suns travel at the same constant angular velocity around the earth over the same distance, but the relative velocity across lines of longitude differs. Initially, the mean sun travelling along the celestial equator would possess a greater velocity across lines of longitude. At the time of the June solstice, the two suns would be aligned and have equal longitude, but then the sun on the ecliptic would travel at a greater angular velocity across lines of longitude.

This key difference in velocity across longitudinal lines between the mean sun and the true sun is vital in correctly explaining the analemma's figure-of-eight shape. The discrepancy between the mean sun and the true sun allows for changes in the length of daylight and changes in sunrise and sunset times. If the sun did not have differing velocities across the lines of longitude over the course of one year, then the position of the sun at the same time of each day would possess the same longitude on the celestial sphere. Similarly, if the sun did not have differing velocities across the lines of latitude, then the latitude of the sun on the celestial sphere would be the same at a specific time of each day. Images of the analemma show varying latitudes depending on time of year.

A three-dimensional diagram using an open celestial sphere resembles that in Figure 5. This figure correctly represents the important knowledge that the path of the mean sun and the path of the true sun are of the same length. Although the diagram is not as simple as the two-dimensional version, it is a more accurate depiction of the true sun's path as compared to that of the mean sun.

By knowing how far 'behind' or 'in front' of the mean sun the true sun is, a conversion can be made to determine the position of the true sun at a specified

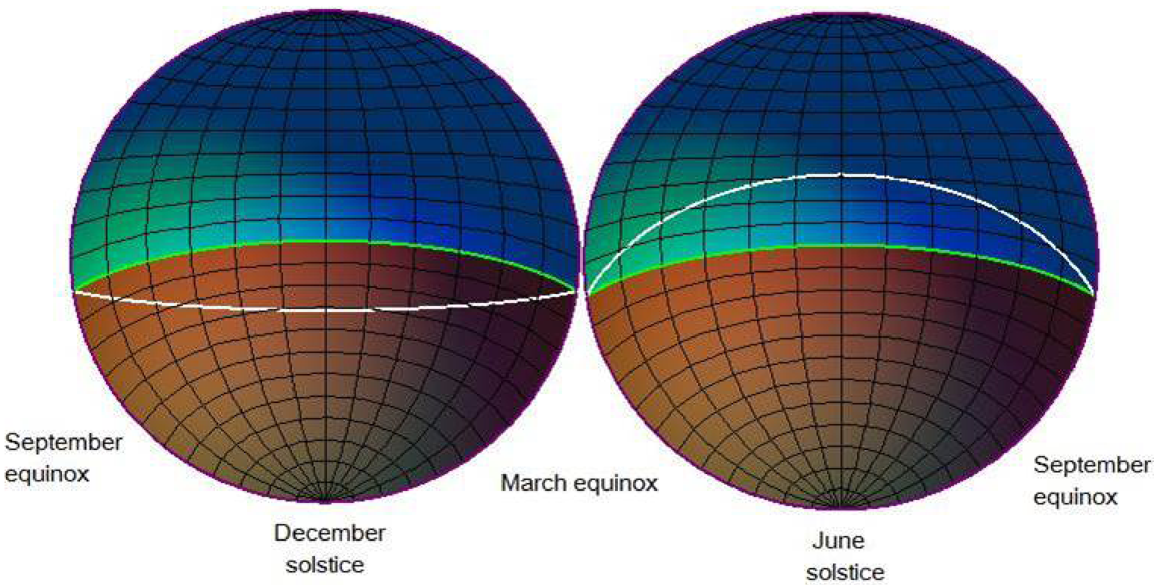


Figure 5. Declination in three dimensions.

moment given the position of the mean sun. The longitudinal velocity of the mean sun is 360 degrees per day or 15 degrees per hour. Using this fact, a relationship between longitudinal difference and time difference between the mean and true suns can be established. The relationship alleviates the need for a three-dimensional graphic, because time, as opposed to longitude, lives on two-dimensional axes.

Equation of Time

Until now, the models of the sun's motion have ignored the fact that the earth's revolution around the sun is elliptical. The physical discrepancy between the true and mean suns has already been graphically represented in the figures of the sun's declination, and the two-dimensional graph of the difference in solar time of a tilted earth and mean solar time naturally has a similar sine curve shape. In the previous section, it was stated that the angular velocity of the mean sun is constant; however, because of the earth's eccentric orbit, the earth must travel at varying speeds in order to maintain a constant angular velocity. The earth's orbit has eccentricity 0.0167, which means it is almost circular. At the perihelion, the earth is closest to the sun and travels fastest; while at the aphelion, the earth is furthest from the sun and travels slowest.

Approximating the location of the sun assuming a circular orbit does not suffice for producing accurate images of the analemma. As with declination, the elliptical orbit can be compared with a simplified circular motion of the mean sun, and the physical discrepancy between these two suns can be expressed in units of time. The time difference between a sun travelling on a circular orbit and a sun travelling on an elliptical orbit also follows a sine curve, where there is no time difference at the perihelion and aphelion.

The total time difference between a mean sun travelling along the celestial equator and the true sun travelling along the ecliptic in an eccentric orbit is given by the equation of time. This is shown in Figure 6. This is simply the summation of the two sine curves derived from the time difference between a sun on the celestial equator and a sun on the ecliptic and the time difference between a circular and elliptical orbit. The equation of time inputs the day of year and outputs time difference between the mean and true sun in units of fraction of a day. It enables a conversion between the position of the mean sun and the position of the true sun.

The analemma is a direct consequence of the equation of time. The deviation in position of the true sun from the mean sun at a given time on each day of the year is provided in the equation of time graph in units of time. It however takes an intuitive leap to be able to mentally transform the equation of time into the analemma, as the relationship is quite complicated.

Figure 7 simulates how the analemma is a physical representation of the equation of time. The equation of time has been scaled to the same axes as the analemma, but the significant points of the equation of time, such as the peaks and zeros, occur at the correct timing. When the equation of time graph is negative, the true sun is moving longitudinally behind the mean sun and the left section of this particular analemma is formed. When the equation of time is positive, the sun is longitudinally ahead of the mean sun and the right portion of this analemma is formed.

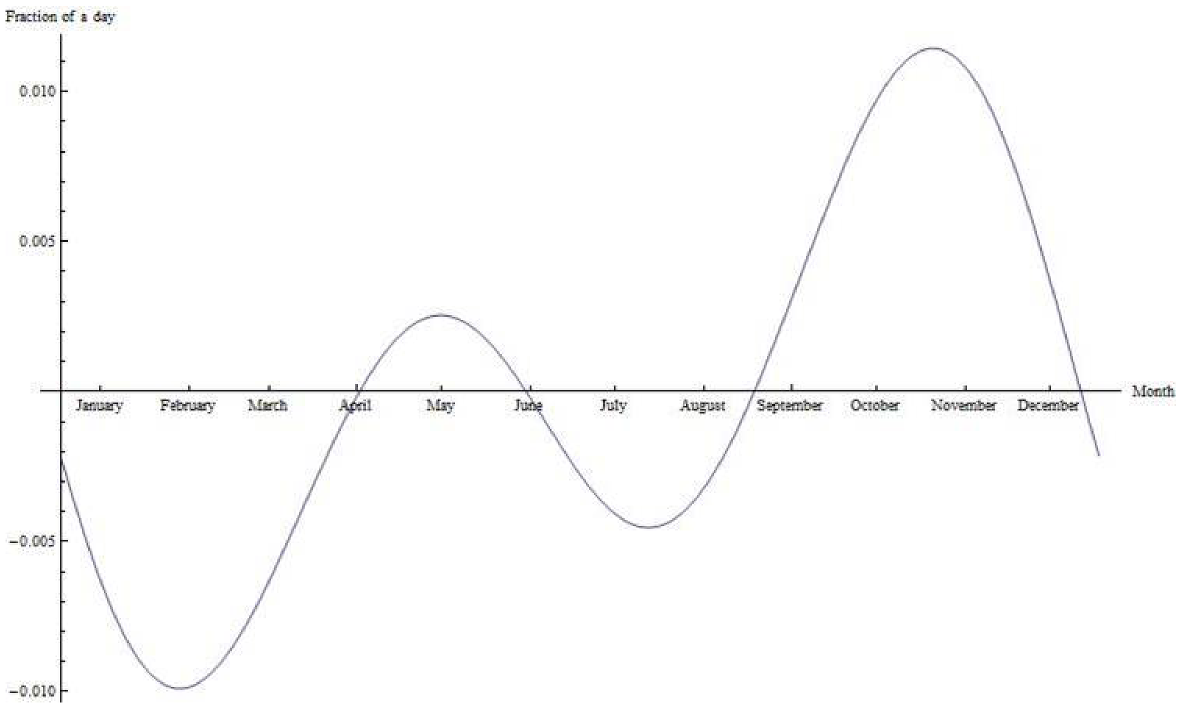


Figure 6. Equation of time.

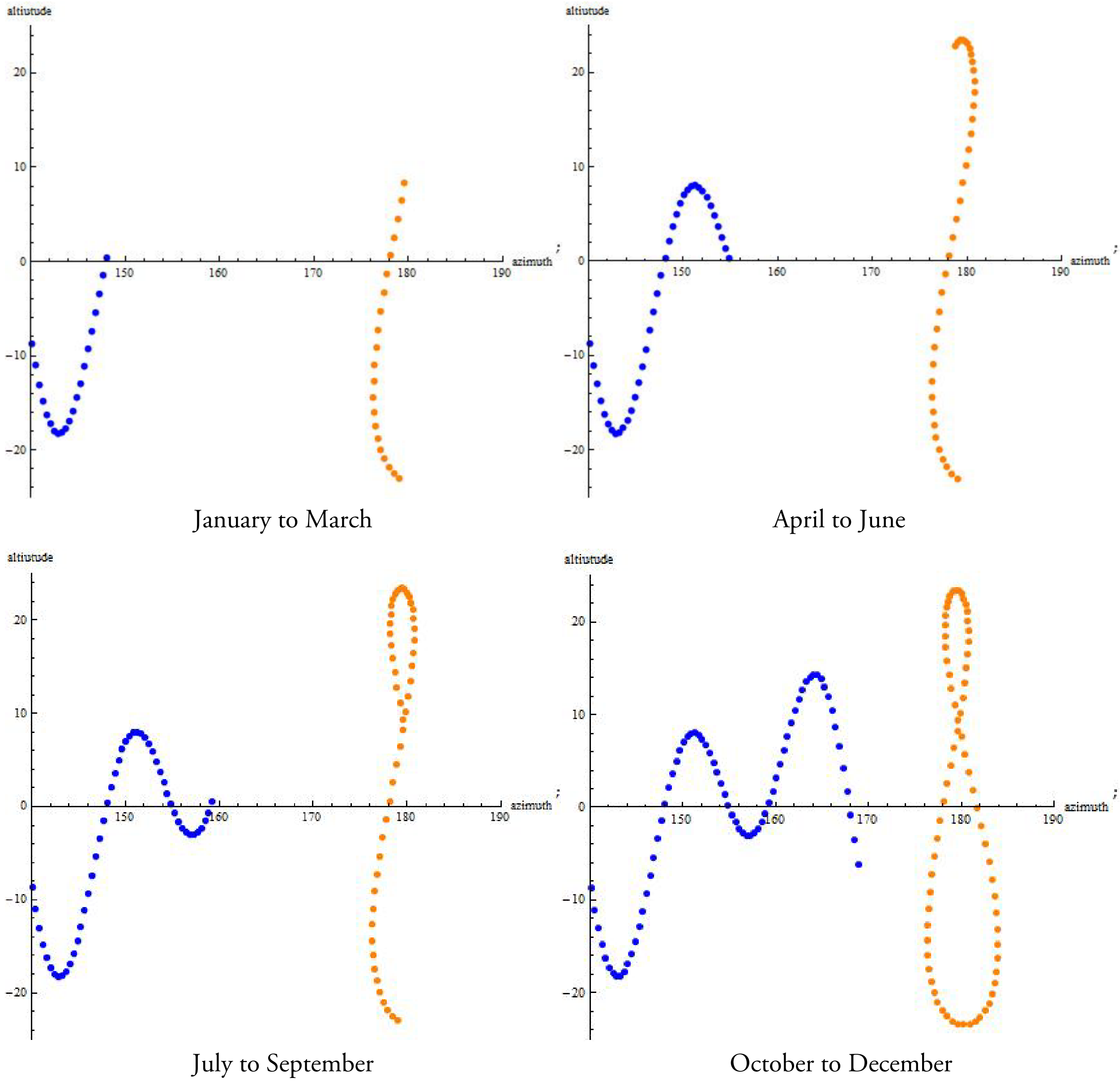


Figure 7. Simulation of a noon analemma and the equation of time.

At this moment, the true sun is seen to be west of the mean sun. The height of the peaks and the depth of the troughs determine the 'width' of each section of the analemma, hence the largest peak and trough correspond to the bottom section of the analemma. This is where there is the greatest time difference between the true and mean suns and so the true sun is longitudinally furthest from the mean sun.

2D Modelling

Two-dimensional models of the analemma involve some form of projection of the analemma as shown on the celestial sphere to a plane. The projection involved in this section maps great circles on the celestial sphere to lines and neither area nor distance is conserved. Azimuth and altitude angles are simply given on a linear scale. Unlike latitude and longitude, azimuth and altitude angles are relative to the position of the observer. Altitude angle ranges from 0 degrees on the horizon to 90 degrees at the zenith and is considered negative if the object is below the horizon. Azimuth angle is determined to be 0 degrees at due north, 90 degrees at due east, 180 degrees at due south and 270 degrees at due west.

To produce figures of the analemma, one must plot altitude against azimuth angle for the position of the sun at each day over the course of a year. Given latitude of the observer ϕ, longitude of the observer λ, declination δ and hour angle H, azimuth angle A and altitude angle h are generated by the proceeding equations (Meeus, 1998) (Teo, 2002).

$$\tan A = \frac{\sin H}{\cos H \sin\phi - \tan\delta\cos\phi}$$

$$\sin h = \sin\phi\sin\delta + \cos\phi\cos\delta\cos H$$

where $\delta = \sin^{-1}(23.45 \sin \lambda)$.

The hour angle H is given by the following equation, where x is the number of hours after noon using mean solar time (12pm in Greenwich).

$$H = 15(x + \text{EquationOfTime[Date]} \times 24)$$

The equation of time outputs time in units of fraction of a day and so this is multiplied by 24 to convert to hours. The summation of mean solar time and the equation of time give the true solar time. This is multiplied by 15 as the sun moves 15 degrees per hour. Here, the equation of time is used to 'correct' the time as measured by a clock (mean solar time) to true solar time, and hence the position of the true sun.

Once the altitude and azimuth of the analemma have been calculated, they can be simply plotted on a two-dimensional plane. However, a similar problem occurs here as when latitude and longitude of geographical locations are plotted on a map, as analemmas found in high altitude angles become extremely distorted.

One useful piece of information gleaned from these figures is the values for altitude and azimuth angle of the analemma. The absolute angular difference between the end of the 'top' loop and the end of the 'bottom' loop of the figure eight shape is always 23.45° + 23.45° = 46.9°. This is because the declination of the sun varies from +23.45° to –23.45° over one year.

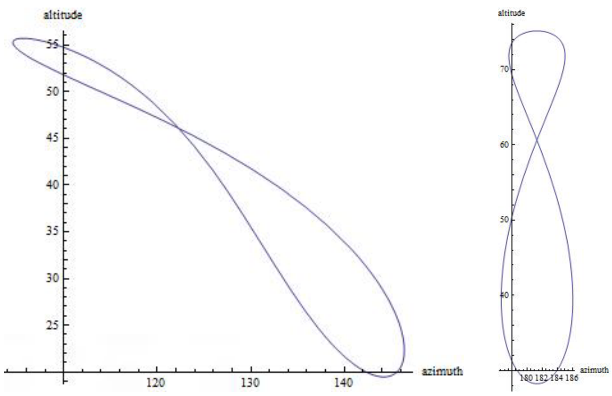


Figure 8: Two-dimensional analemmas at 38°18'N.

3D Modelling

A parametrisation of a sphere can be given by:

$$\begin{aligned} x &= r\cos v \cos u \\ y &= r\cos v \sin u \\ z &= r \sin v \end{aligned}$$

where r is the radius of the sphere and $u, v \in [0, 2\pi)$.

Using this parametrisation, the two-dimensional models in the previous section can be placed on a sphere of radius r = 1, where v is the altitude angle h, and u is the azimuth angle A at a given time t, latitude ϕ and date D.

It then follows that a parametrisation for the analemma curve at a given time t and latitude ϕ would be:

$$\begin{aligned} x &= \cos(h(t,\phi,d))\cos(A(t,\phi,d)) \\ y &= -\cos(h(t,\phi,d))\sin(A(t,\phi,d)) \\ z &= \cos(h(t,\phi,d)) \end{aligned}$$

where $D \in [D_0, D_0 + 365]$.

This parametrisation will be used to create three-dimensional models by projecting the position of the sun as given in azimuth and altitude angles onto a sphere.

Astronomical maps and star charts generally present the northern or southern skies from a viewpoint at the opposite pole on the celestial sphere. This is practical because an entire hemisphere can be seen in one image while the azimuth and altitude angles of celestial bodies can easily be read from the chart. The analemma can also be presented in the manner in order to represent its position in the sky for a viewer at a given latitude. In Figure 9, the hemisphere is concave and so we are

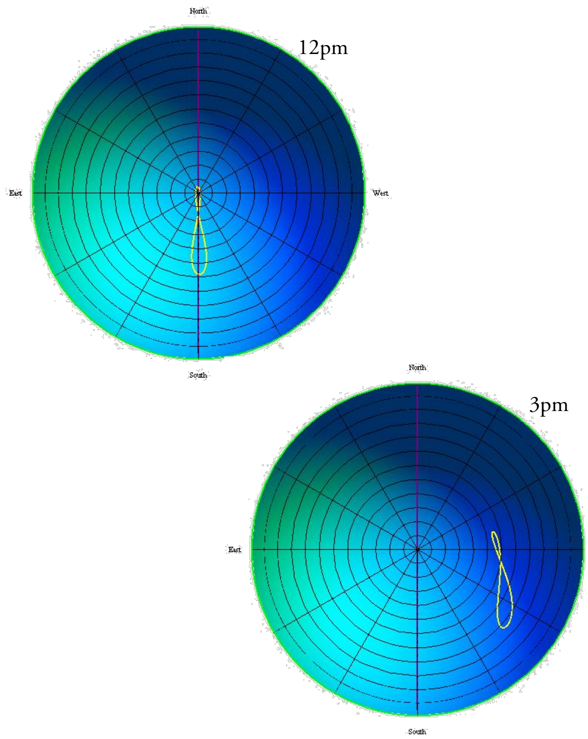


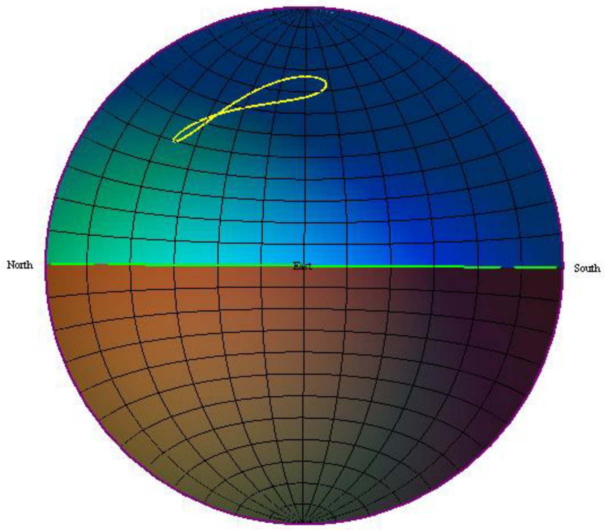
Figure 9: View from below at 20°N.

viewing the sky from the 'inside'.

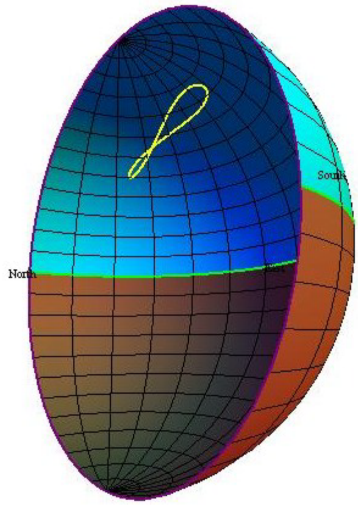
These images are beneficial as they give a clear indication of where the analemma is to be found in the sky at a certain time of day. It is important to notice that east is on the left-hand side because the observer is looking upwards at the sky, not downwards onto a map. In the above right figure, the analemma is located in the west, and so it is an afternoon analemma (in this case 3pm). We see that the sun spends most of the year in the southern sky, and so the observer must be in the northern hemisphere (in this case 20 degrees north).

A significant problem with such graphics is that the viewing point is from the opposite end of the celestial sphere. The human eye cannot see a complete 360-degree view whilst focusing on the zenith. Creating such an image in Mathematica is only possible by taking the viewpoint to be a location outside the interior region of the celestial hemisphere, in this case at the opposite pole of the celestial sphere. It is quite unnatural to look upwards at the sun during the daytime in this manner, as usually we notice the sun with respect to the horizon. In these graphics, points below the horizon are simply omitted and so there is little scope for depiction of events such as sunset and sunrise. When the sun nears the horizon, the skewing of images becomes most drastic and therefore such a viewing point is only practical when the sun is overhead.

By dissecting the celestial sphere through the zenith, the horizon becomes better depicted graphically and can be used as a reference point. This seems to be a more natural depiction of the analemma. Such images may be most useful for explaining the relationship between the analemma and sunrise and sunset. As this analemma is viewed from a latitude in the southern hemisphere, the figure eight shape is inverted.



10am view from the horizon at 20°S

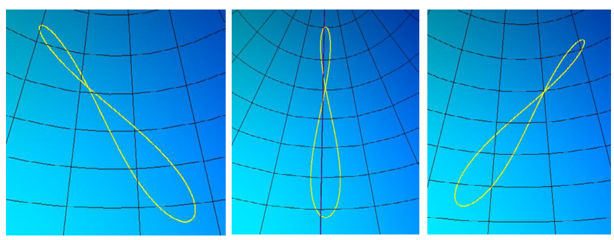


Rotated 10am view at 20°S

Figure 10: Analemma viewed from different perspectives on the celestial sphere.

In the three-dimensional hemispherical model on the left of Figure 10, distortion occurs along the boundary of the hemisphere, where the analemma is furthest from the viewing centre. This is a result of the optical distortion of objects far from the point of focus. The only way to view an object correctly is to look directly at it. By looking directly at the analemma, as shown in the below figure, the analemma appears less distorted. Although this viewing point is still inaccurate, because it is at a location on the celestial sphere and not on the earth, it significantly improves the appearance of the shape of the analemma. The analemma appears relatively straight and the distortion evident in the left figure is corrected.

The following series of images in Figure 11 show the analemmas as they would be viewed from the earth. The view vector is from the centre of the sphere (the earth) to the centre of the analemma (position of the mean sun). These graphics of the analemma bear the strongest resemblance to photographs of the analemma taken from earth and are both realistic and accurate. The azimuth and altitude angles of the mean sun are marked in brackets at the location of the mean sun at the given time and latitude.



9am 12pm 3pm

Figure 11: Corrected viewpoint at 30°N.

Case Studies

Sunrise in Singapore

Correct and realistic viewing of the analemma assists in explaining related phenomena accurately such as the varying sunrise times in Singapore. As Singapore lies near the equator (1°22'N), an early morning analemma appears horizontal. Sunrise times in Singapore range from around 6:45am to 7:15am, although there is only about eight minutes variation between the length of the shortest and longest day (Aslaksen, 2012). Figure 12 shows the shape of the analemma at 7am in Singapore. Azimuth and altitude angles are marked in order to estimate the width and the height of the analemma.

The angular distance between the highest and lowest azimuth angle of the analemma is approximately 47 degrees (23.45° + 23.45°). The angular distance between highest and lowest altitude angle of the analemma is approximately 7.5°. The sun travels 15 degrees every hour, so the 7.5-degree variation between the highest and lowest altitude angle on the analemma implies a 30-minute variation in sunrise time. The points of earliest and latest sunrise on the analemma correspond to around November 3 and February 10 respectively.

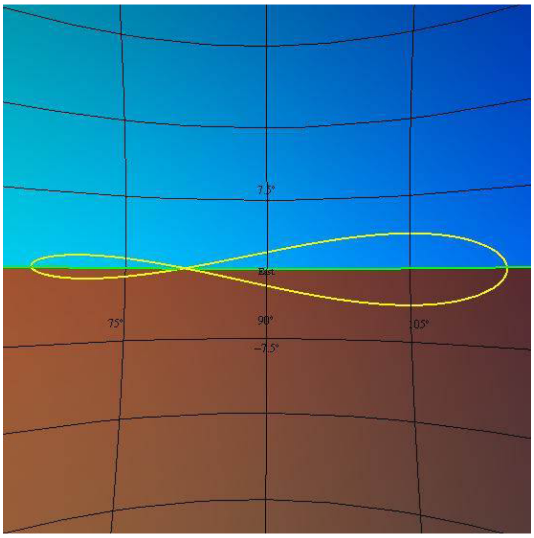


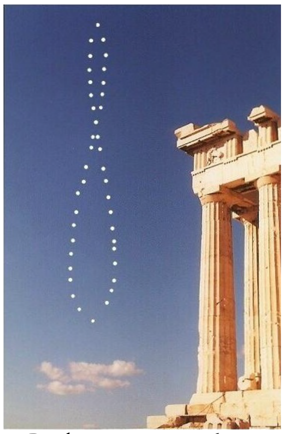
Figure 12: Sunrise in Singapore.

The Analemma in Athens

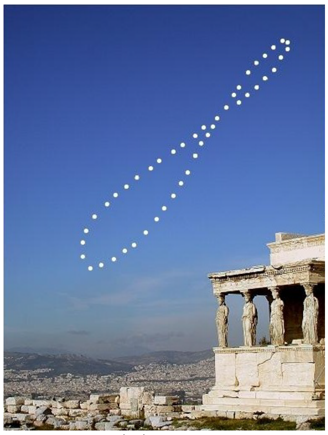
In order to photograph the analemma, a camera must be carefully positioned such that the camera's frame is wide enough to capture all possible locations of the sun at a particular time of day. The sun is then captured at a specific time of day over a period of one year. A vertical analemma occurs at mean solar noon, the mean time of when the sun crosses the celestial meridian over one year. Photographer Anthony Ayiomamitis has captured numerous images of the analemma in Athens, which are depicted in Figure 13.

In the above centre figure, Ayiomamitis attempted to capture a perfectly straight analemma occurring at mean solar noon. Unlike in Greenwich, mean solar noon does not occur at around 1200, rather at some time between 1200 and 1300, due to Athens' position within a time zone boundary.

Hephaisteion, 10am



Parthenon, 12:28.16pm



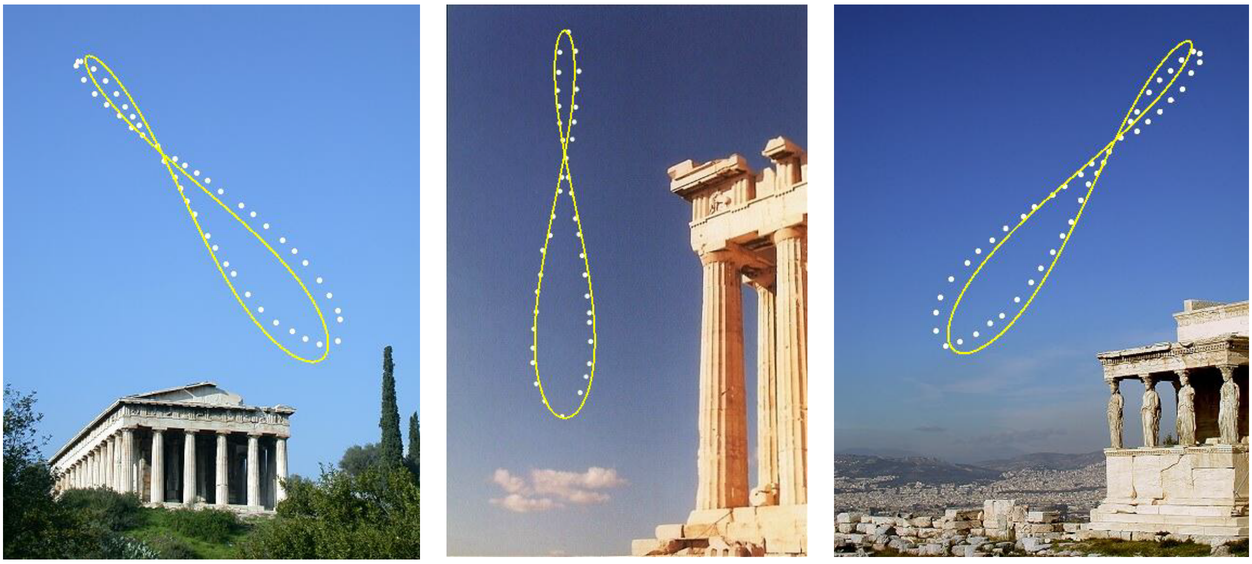
Erechtheion, 3pm

Image Data

Photographer: Anthony Ayiomamitis
Latitude: 38.29997°N
Longitude: 23.7430°N
Time zone: UTC+2
Dates recorded: January 7, 2003 to December 20, 2003 (10am and 3pm analemma), January 12, 2002 to December 21, 2002 (12:28.16pm analemma)

Figure 13: The Analemma in Athens (Ayiomamitis, 2001-11).

The earth rotates at an average velocity of 1/15 hours per degree. The photographs are recorded to have been taken at 23.7340 degrees longitude, so the actual time zone of Athens should be 24/360 × 23.7340 = +1.583 hours. As Athens is technically in UTC+2, 2 – 1.583 = 0.417 hours or 25 minutes and 2 seconds should be subtracted from times given under the time zone UTC+2. Mean solar noon in Athens should occur at 12:25.2pm UTC+2. Interestingly, Ayiomamitis' photograph of the noon analemma was taken at 12:28.16pm UTC+2. The reason behind this is that the position of the analemma in the sky varies annually. The Gregorian calendar year approximates the time taken for the earth to revolve around the sun and every four years, a correction is in the form of a leap year. A more accurate way to calculate the mean solar time for a particular location in a particular



Hephaisteion, 10am — Parthenon, 12:28.16pm — Erechtheion, 3pm

Figure 14. Model testing.

year is to calculate the time of day where the sun at the summer solstice and winter solstice would have the same azimuth angle. Indeed this is the method used by Ayiomamitis, and he found this to occur at 12:28.16pm.

After corrections are made for the time zone in which the images were taken, models made using Calendrica should strongly resemble the photographs of the analemma. Specifically, the shape and angle of the analemma in the models and the photographs should be the same, if the camera in the photographs is approximately pointing towards the position of the mean sun. They are depicted in Figure 14.

Although the photograph of the vertical analemma is replicated almost perfectly in the model, the morning and afternoon analemma models are both positioned at greater angles to the horizon than the photographs of the analemma at these times. There seems to be some systematic error in the assumptions made when creating the models. The accuracy of the noon analemma implies that the time corrections and the longitude of the photographer are correct. If the photographer were at a higher latitude, then the analemma would possess a greater angle at 10am and 3pm. The latitude provided by the photographer is, however, likely to be correct. A much more likely issue in matching the models to the photographs is the line of sight of the camera. If the camera is not positioned towards the mean sun, then numerous visual effects can affect the shape and angle of the analemma. The camera presumably corrected for visual distortions by using a rounded lens, but it is difficult to determine how the camera may change the angle of the analemma.

We can direct the viewing vector in the model at another point in the sky so that the analemma appears at the correct angle. However, we can also change the viewing vector such that the analemma appears in many desired forms. To proceed with modifying the viewing direction in the programmatic figure is not very useful unless more information is acquired about the direction in which the camera is pointing and the camera's mechanisms for correcting visual distortions.

In Figure 15, the 'X' marks the location of the direction of the adjusted viewing vector. By shifting the angle of the viewing vector to some point a few degrees east of the mean sun in the above left figure and to some azimuth angle a few degrees west of the mean sun in the above right figure, the angle of the analemma changes in such a way that it approximately matches the photographs. This does not necessarily mean that Ayiomamitis' camera was pointed in these directions, as a camera lens may also adjust the angle of the analemma.

Conclusion

Many astronomical phenomena can be explained with typical models, such as the celestial sphere as

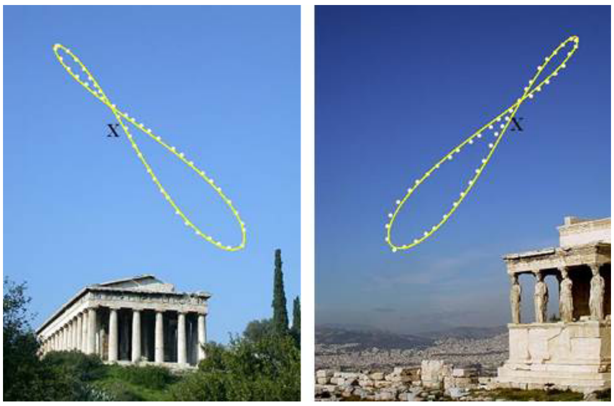


Figure 15: Adjusting the viewing direction.

seen from an external perspective or two-dimensional projections. An external view of the celestial sphere is useful for understanding orbital motions, such as the revolution of the earth around the sun, whereas the declination of the sun at different times throughout the year can be better depicted from inside the celestial sphere. When projecting the celestial sphere onto a plane, different projections should be used depending on whether lengths, area, or shape should be conserved. Often it is simpler and more accurate to depict three-dimensional phenomena on three-dimensional axes. The way in which objects appear depends on the viewer's line of sight. In order to minimise visual distortions and see an object correctly, it is essential to look directly at the object.

By modelling the analemma using programmable figures, it is possible to see how the analemma appears at different parts of the world at different times of the day. There are various ways to depict the analemma, but the most useful way to depict the analemma is realistically and accurately. The analemma is a three-dimensional phenomenon seen from the earth. Good models of the analemma must therefore be presented on three-dimensional axes and be viewed from the centre of the celestial sphere. View point and line of vision can significantly affect how the analemma appears in programmatic figures, even when the model is scientifically correct. Realistic and accurate models of the analemma should resemble photographs of the analemma. Through comparing models of the analemma in Athens with Ayiomamitis' photography, the shape of the models is shown to be accurate.

The angle of the analemma in the models closely matches that in the photographs, in which the discrepancy is possibly due to the inability to view the models in the same direction in which the camera is pointing. The appearance of the analemma is dependent on the location and line of sight of the viewer, be it from outside, on or inside the celestial sphere. In this project, the most useful depictions were those that viewed the centre of the analemma from a viewing point on the earth. These models closely reflect how we would see the analemma from our location and assist in explaining the irregular path of the sun at the same time of each day throughout each year.

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